

ARTICLES

THE ROLE OF SECTORAL SHIFTS IN THE DECLINE OF REAL GDP VOLATILITY

DANIEL BURREN AND KLAUS NEUSSER

University of Bern

U.S. production has shifted from goods-producing to service-producing industries. We assess whether this shift contributed to the decline in U.S. output volatility over the period 1949–2005 and provide an estimate of its relative importance. Growth rates of GDP by industry are analyzed in a seemingly unrelated multivariate autoregression framework with time-varying innovation covariance matrices. These changing unobserved covariance matrices are modeled as a Wishart autoregressive process of order one, which results in a nonlinear state-space system. The particle filter is used to obtain estimates of the innovation covariance matrix at each point in time. Several counterfactual experiments make it possible to apportion the decline in output volatility between the shift in the sectoral composition and changes in innovations. Our main finding is that the shift into the service sector can explain about 30% of the decline in GDP's volatility, despite the fact that some sectors became even more volatile. This result is robust across a wide variety of alternative specifications.

Keywords: Sectoral Shifts, Great Moderation, Stochastic Volatility, Wishart Autoregressive Process, Particle Filter

1. INTRODUCTION

The observed decline in U.S. output volatility over the post-WWII period, commonly known as the Great Moderation, triggered a large empirical literature.¹ Although the decline is well documented and widely accepted, its exact nature still remains controversial. On one hand, Blanchard and Simon (2001) argue for a steady decline over the period 1950 to 2001. They relate the gradual decay to decline in the volatility of government spending, consumption, and investment and to a change in the sign of the correlation between inventory investment and sales. Furthermore, they find that increased inflation volatility took output volatility temporarily off its declining trend in the 1970s and early 1980s. Blanchard and Simon (2001) state that demand side-shifts have little to do with the general evolution

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of output volatility. On the other hand, Stock and Watson (2005b) and Fang and Miller (2008) present evidence for a sharp break around 1984. They attribute this to a combination of improved policy (20 to 30%), good luck in the form of reduced commodity price shocks (20 to 30%), and further nonidentifiable elements. More recently, Galí and Gambetti (2009) have presented a more complex picture that goes beyond the pure good luck hypothesis. They point to some deeper underlying structural changes in the U.S. economy.

This paper focuses on the structural changes in the sectoral composition of the U.S. economy and establishes the contribution of these changes to the explanation of the Great Moderation. Not only do we present a more detailed characterization of the volatility decline in terms of timing, but we also propose a new device for modeling time-varying variance–covariance matrices.

More specifically, we investigate the evolution of U.S. NAICS GDP by industry data in two stages. In a first stage, we estimate an autoregression of order one, AR(1), for each sectoral growth-rate series. We check the stability of the regression parameters and the innovation variances against the alternative of an unknown break date. Interestingly, we find that the innovation variance increased for the mining, utilities, and information sectors, but decreased only for the durable goods sector. This goes against the Good Luck hypothesis, which attributes the Great Moderation to a decline in the magnitude of shocks [see Stock and Watson (2005b)]. We then use the estimated AR(1) processes to run counterfactual Monte Carlo experiments. Drawing the innovation errors from a multivariate normal distribution with covariance matrix corresponding to the pre-1984 (post-1984) period, we establish that sectoral shifts explain approximately 30% (15%) of the decline in output volatility.

In the second stage, we allow the covariance matrix of the innovations to change smoothly over time. More precisely, we assume that the covariance matrix of the innovations follows a Wishart autoregressive process of order one (WAR(1)). This process has been introduced by Gouriéroux et al. (2009) to model multivariate stochastic volatility. It has the property that it naturally accommodates the positiveness and symmetry of covariance matrices.² Although these features are also shared by other approaches [see for example Primiceri (2005)], the WAR representation has the advantage that it can capture a wide range of smooth evolutions in a parsimonious way. Moreover, it makes it possible to write the multivariate process of sectoral growth rates as a nonlinear state-space system, with the covariance matrix being the unobserved state. The nonlinearity of the system precludes the use of the Kalman filter to infer the unobserved state. The particle filter, however, still provides a valid alternative procedure that makes it possible to estimate the unobserved covariance matrices. These estimates can then be used to run further counterfactual exercises. Again we find that structural shifts, particularly in favor of the service sector, can account for more than one-fifth of the Great Moderation.

The paper is structured as follows. Section 2 presents the dataset and some preliminary findings. These results allow a first corroboration of our hypothesis. Section 3 reviews the Wishart process and the particle filter, together with the

implementation of the estimation procedure. Section 4 presents the empirical results. Section 5 concludes the paper.

2. DATA AND PRELIMINARY RESULTS

2.1. Data Description

We use two-digit annual U.S. GDP by industry series from the Gross-Domestic-Product-by-Industry Accounts (Bureau of Economic Analysis) ranging from 1948 to 2005. These series sum up to nominal GDP. They were deflated using the implicit GDP deflator³ and expressed in per capita terms using the civilian noninstitutional population index from the Bureau of Labor Statistics. The dataset consists of 22 sectors, listed in Table 1. Except when noted otherwise, growth rates are computed in the usual way as

$$100 \frac{X_t - X_{t-1}}{X_{t-1}}.$$

As a first check of our hypothesis, we plot the average sectoral growth rates against their coefficient of variation.⁴ The corresponding Figure 1 clearly reveals a negative relationship with a correlation coefficient of -0.47 . Thus, less volatile sectors have grown faster than more volatile ones. This implies that the growing share of low-volatility sectors led to a decline in aggregate volatility, i.e., the volatility of GDP growth. The aim of our paper is to quantify this effect and check how much it contributes to the Great Moderation.

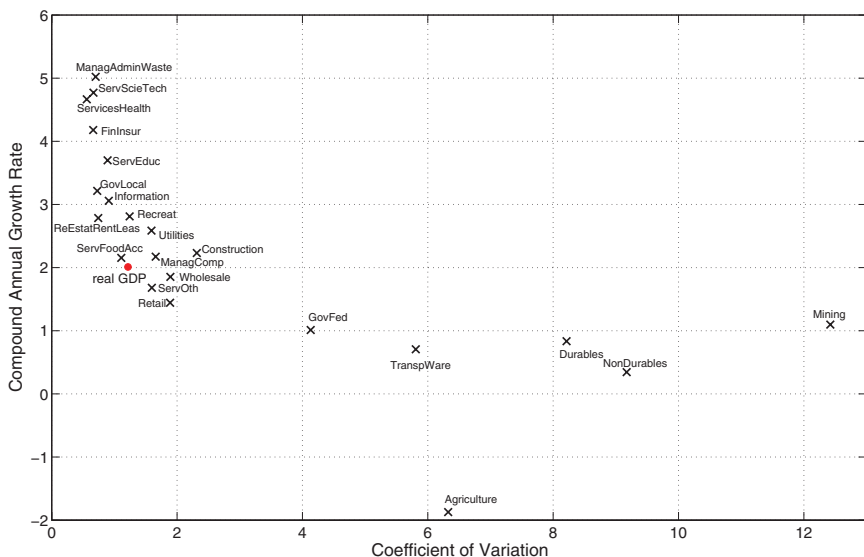


FIGURE 1. Growth and coefficient of variation.

TABLE 1. Sectors with some summary statistics

Sector	Abbreviation	Share in		Compound annual growth rates	Std of growth rate
		1950	2005		
Agriculture—forestry— fishing—hunting	Agriculture	6.8	0.9	−1.87	11.85
Mining	Mining	2.6	1.7	1.10	13.63
Utilities	Utilities	1.6	1.9	2.58	4.12
Construction	Construction	4.4	4.8	2.23	5.16
Durable goods	Durables	14.8	7.0	0.83	6.85
Nondurable goods	NonDurables	12.2	5.0	0.34	3.15
Wholesale trade	Wholesale	6.3	5.9	1.85	3.51
Retail trade	Retail	8.8	6.6	1.44	2.73
Transportation and warehousing	TranspWare	5.9	2.9	0.71	4.10
Information	Information	2.3	4.6	3.06	2.78
Finance and insurance	FinInsur	2.7	8.1	4.18	2.77
Real estate—rental and leasing	ReEstatRent	8.7	12.5	2.78	2.07
Professional—scientific and technical services	ServScieTech	1.6	6.9	4.77	3.18
Management of companies and enterprises	ManagComp	1.7	1.8	2.17	3.61
Administrative and waste management services	ManagAdminWaste	0.6	3.0	5.02	3.53
Education services	ServEduc	0.4	0.9	3.70	3.30
Health care and social assistance	ServicesHealth	1.7	6.9	4.67	2.61
Arts—entertainment and recreation	Recreat	0.6	0.9	2.81	3.49
Accommodation and food services	ServFoodAcc	2.4	2.7	2.15	2.39
Other services without government	ServOth	2.8	2.4	1.68	2.68
Federal government	GovFed	6.2	4.0	1.01	4.18
State and local government	GovLocal	4.6	8.5	3.21	2.33
Real GDP per capita				2.01	2.45

Note: Sample period: 1949—2005.

2.2. Univariate Analysis of Sectoral Growth Rates

Before proceeding to the multivariate analysis that is the focus of our investigation, we exploit the univariate time series properties of sectoral growth rates. This serves two purposes: First, we want to prepare the ground by common and

comprehensible techniques and persuade the reader that there is indeed a scope for deeper investigation; second, the process provides some indication of the appropriate multivariate specification.

We model each sectoral growth rate as a simple autoregressive processes of order one (AR(1) process), but allow structural breaks in the coefficients at some unknown date:

$$x_t^i = [1 - d_t(\tau)]c_1^i + d_t(\tau)c_2^i + [(1 - d(\tau)_t)\phi_1^i x_{t-1}^i + d_t(\tau)\phi_1^i x_{t-1}^i + \varepsilon_t^i, \\ \varepsilon_t^i \sim \text{MDS}(0, \sigma^2(\varepsilon_t^i))], \quad (1)$$

where x_t^i denotes the growth rate of sector i . $d_t(\tau)$ is a dummy variable variable that takes the value zero for $t < \tau$ and one for $t \geq \tau$. τ denotes the (unknown) date of the break point. The innovations ε_t^i are assumed to be martingale differences sequences (MDS) with possibly time-varying variances $\sigma^2(\varepsilon_t^i)$. The dummy variable effectively breaks the sample into two parts. A simple AR(1) model with constants c_1^i and autoregressive coefficients ϕ_1^i , respectively c_2^i and ϕ_2^i , is estimated for each subsample.⁵

Testing structural breaks in mean. We assess the null hypothesis of constant parameters against the alternative of a single unknown break time by the Andrews-Ploberger test [see Andrews and Ploberger (1994)]. The corresponding test statistic is given by

$$\xi_W = \sup_{[0.15T] \leq \tau \leq T - [0.15T]} W(\tau),$$

where $W(\tau)$ denotes the Wald test statistic for the joint null hypothesis $c_1 = c_2$ and $\phi_1 = \phi_2$ given a break point at date τ . The appropriate test statistics is then obtained by taking the supremum over all $W(\tau)$, where τ varies in a given time interval. The time interval is set $[[0.15T], [T - 0.15T]]$, where T denotes the sample size and 0.15 is the recommended trimming factor. The critical values are provided by Andrews and Ploberger (1994). Note that they remain valid even for heteroskedastic residuals as long as a robust estimator is used for the covariance matrix in the Wald statistic. The value $\tau^* = \arg \max_{\tau} W(\tau)$ provides an estimate of the break period.

The evidence presented in Table 2 shows significant breaks (at the 5% level) for mining, utilities, retail trade, real estate—rental and leasing, professional—scientific and technical services, education services, health care and social assistance, accommodation and food services, and state and local government, but none for agriculture and federal government. The F -statistics indicates that an AR(1) model with possibly changing coefficients is sufficient to characterize the sectoral growth rates. We have omitted the results for the agricultural and federal government sector in Table 2 because they fall outside the AR(1) framework of equation (1).

TABLE 2. Andrews–Ploberger parameter stability test

Sector	\hat{c}_1^i	\hat{c}_2^i	$\hat{\phi}_1^i$	$\hat{\phi}_2^i$	$\hat{\mu}_1$	$\hat{\mu}_2$	max W	F -stat	τ^*
Mining	2.18 (1.81)	1.02 (3.35)	0.30 (0.17)	0.13 (0.16)	3.11 (5.69)	1.17 (3.96)	10.11	2.29	1956
Utilities	3.73 (2.15)	1.71 (0.79)	0.33 (0.20)	0.09 (0.20)	5.58 (10.39)	1.88 (1.17)	17.74	0.32	1987
Retail	1.35 (0.64)	0.81 (0.56)	0.15 (0.17)	0.35 (0.27)	1.60 (0.83)	1.25 (0.50)	13.23	0.06	1960
ReEstatRentLeas	1.24 (0.47)	2.93 (0.91)	0.56 (0.12)	−0.31 (0.31)	2.79 (0.82)	2.23 (1.54)	20.46	0.21	1986
ServScieTech	8.18 (1.60)	2.95 (0.76)	−0.21 (0.23)	0.35 (0.13)	6.75 (9.57)	4.55 (2.78)	24.95	0.36	1967
ServEduc	1.18 (0.59)	3.01 (1.21)	0.67 (0.10)	0.21 (0.25)	3.54 (1.46)	3.79 (3.51)	20.58	0.22	1977
ServHealth	9.21 (1.64)	2.25 (0.65)	−0.55 (0.22)	0.43 (0.16)	5.96 (8.96)	3.97 (1.67)	26.35	0.17	1992
ServFoodAcc	2.25 (0.60)	1.83 (0.38)	−0.73 (0.23)	0.26 (0.07)	1.30 (0.68)	2.46 (0.87)	22.43	0.01	1962
GovLocal	0.73 (0.39)	1.02 (0.46)	0.77 (0.09)	0.36 (0.24)	3.23 (0.75)	1.60 (0.28)	64.42	1.01	1956

Note: White’s robust standard deviations are in parentheses. max W is the maximum Wald statistic from the Andrews test; the 5% critical value is 8.68. The F -stat tests the significance of remaining autocorrelation in residuals up to lag two; its theoretical 95th quantile is 3.18. τ^* is an estimate of the break period. The estimated mean growth rate for sector i is $\hat{\mu}^i = \hat{c}^i / 1 - \hat{\phi}^i$ and its standard deviation is obtained using the delta method.

Testing structural breaks in variance. We next examine the evolution of innovation variances. According to the Good Luck hypothesis, economic fluctuations have moderated because of less volatile shocks. If this is true, we should see a decline in the innovation variances of sectoral growth rates; i.e., the variances of the ε_t^i should have decreased. We investigate this hypothesis using a test proposed by Inclán and Tiao (1994) that is based on an iterated cumulative sums of squares (ICSS) algorithm. We apply the test to the residuals from the AR(1) regressions (see equation (1)). We replace model (1) with a simple AR(1) model if the Andrews–Ploberger test does not detect a break in the parameters.

The evidence reported in Table 3 shows significant one-time breaks in the variances of mining, utilities, durable goods, and information with break dates

TABLE 3. Variance break test

Sector	Break date τ	$\sigma(\varepsilon_{t < \tau}^i)$	$\sigma(\varepsilon_{t \geq \tau}^i)$	$\sigma(x_{t < \tau}^i)$	$\sigma(x_{t \geq \tau}^i)$
Mining	1973	5.66	16.47	5.92	16.84
Utilities	1973	2.10	4.19	3.46	4.14
Durables	1959	11.22	5.74	9.50	6.02
Information	1995	2.30	4.32	2.36	4.15

1973, 1973, 1959, and 1995, respectively.⁶ Table 3 also reports the estimated conditional and unconditional standard deviations before and after the break date.

Note that the innovation variances of mining, utilities, and information actually increased, which goes against the wisdom that GDP growth has become more stable thanks to less volatile economic shocks. In addition, there seems to be no relation between parameter breaks and conditional variance breaks. Only for mining and utilities there is evidence for breaks in both parameters and innovation variance. The break dates, however, lie several years apart.

Inclán and Tiao (1994) point out that their test has low power in samples with less than 100 observations if the change in variance is small. Therefore, we take 1984 as the potential break date. This corresponds to the break date indicated by the ICSS test for GDP growth and is also proposed by Stock and Watson (2005b). If the Good Luck hypothesis is true, the innovation variances of the sectoral growth rates should have dropped at the same time as the variance of GDP growth. As the break date is known, we can use the Goldfeld–Quandt test. The test statistic is given by

$$F = \frac{\hat{\sigma}^2(\varepsilon_{t < 1984}^i)}{\hat{\sigma}^2(\varepsilon_{t \geq 1984}^i)} \sim F_{n_1 - K, n_2 - K},$$

where ε_t^i denotes the corresponding residuals. Under the null of equal variances the test statistic has a F -distribution with $n_1 - K$ and $n_2 - K$ degrees of freedom (n_i being the number of observations in subsample i and K being the number of regressors). Because the alternative hypothesis is a decrease in sectoral innovation variances, the 5% critical value is given by the 0.95 quantile, which is 2.04 in the present case.

The results in Table 4 indicate that only the variances for durable goods, non-durable goods, retail trade, transportation and warehousing, and state and local government decreased significantly, not the variances for the remaining sixteen sectors.⁷ This evidence therefore does not support the Good Luck hypothesis. However, we conclude that changes in the innovation variances indeed took place.

Sectoral shifts and output volatility. We end this preliminary investigation by providing some first evidence of the role of sectoral shifts in output volatility. Denote by ζ_t the vector of sectoral shares, defined as the ratio of sectoral real value added over total real GDP, and by \hat{W} the sample covariance matrix of the residual vector $\hat{\varepsilon}_t = (\hat{\varepsilon}_t^1, \dots, \hat{\varepsilon}_t^{22})'$. Then $\zeta_t' \hat{W} \zeta_t$ can be considered as an estimate of real GDP volatility taking the change in the sectoral composition into account. This entity declined steadily from almost 2.9 in the 1950s to 2.1 in 2005, as shown in Figure 2. This finding corroborates our hypothesis. The analysis is, however, incomplete because the covariance matrix W is not constant over time. We account for this fact in the following sections.

TABLE 4. Standard deviations and Goldfeld–Quandt structural break test with fixed break date 1984

Sector	$\sigma(\varepsilon_{t<1984}^i)$	$\sigma(\varepsilon_{t\geq 1984}^i)$	$\sigma(x_{t<1984}^i)$	$\sigma(x_{t\geq 1984}^i)$	F-stat
Agriculture	11.42	11.21	11.09	13.12	1.04
Mining	11.31	16.03	11.61	16.25	0.50
Utilities	3.71	2.91	4.36	3.11	1.63
Construction	5.09	4.27	5.37	4.87	1.42
Durables	8.03	4.67	8.04	4.61	2.95*
Nondurables	3.62	2.17	3.66	2.27	2.79*
Wholesale	3.81	2.94	3.92	2.87	1.68
Retail	3.12	1.90	2.85	2.54	2.70*
TranspWare	4.72	2.84	4.68	2.92	2.76*
Information	2.36	3.34	2.33	3.41	0.50
FinInsur	2.56	2.35	3.02	2.40	1.18
ReEstRentLeas	1.83	1.63	2.26	1.77	1.60
ServScieTech	2.81	3.15	2.82	3.71	0.80
ManagComp	3.72	3.51	3.64	3.55	1.12
ManagAdminWaste	3.12	3.77	3.06	4.18	0.69
ServEduc	2.80	2.01	4.00	1.95	1.96
ServHealth	2.17	2.10	2.62	2.36	1.06
Recreat	3.06	3.99	3.00	3.90	0.59
ServFoodAcc	2.40	1.89	2.59	2.05	1.61
ServOth	2.87	2.17	2.73	2.45	1.74
GovFed	4.85	2.19	5.06	2.35	4.89*
GovLocal	1.44	0.80	2.65	1.08	3.26*

Note: The theoretical 95th quantile of the F -distribution for testing the equality of the variance of ε_t^i is 2.04.

3. SMOOTHLY CHANGING COVARIANCE MATRIX

In contrast to Stock and Watson (2005a) and Fang and Miller (2008), Blanchard and Simon (2001) argued that U.S. output volatility did not drop suddenly around 1984, but declined steadily over time. As a consequence, the covariance matrix of the AR(1) innovations should have changed steadily too. We investigate this hypothesis by modeling the covariance matrix as a latent variable with an autoregressive structure of order one. In order to respect the character of a covariance matrix, we follow Gouriéroux et al. (2009) and model the covariance matrix as a Wishart autoregressive process of order one (WAR(1)).

3.1. The Wishart Autoregressive Process

The sectoral growth rates $x_t = (x_t^1, \dots, x_t^{22})'$ are assumed to follow an AR(1) process as described. However, we allow the covariance matrix of the innovations,

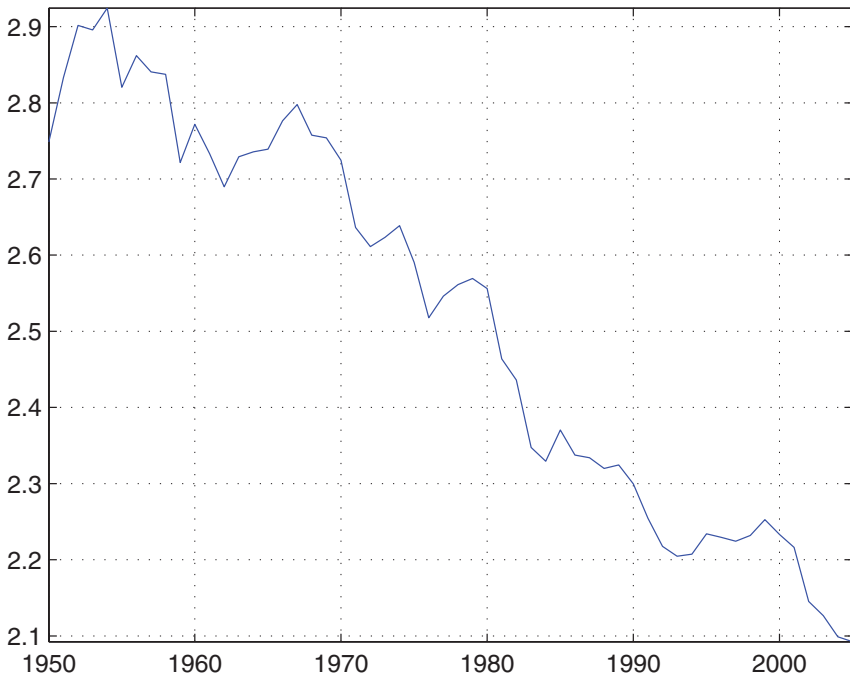


FIGURE 2. Volatility of GDP as implied by sectoral volatility ($\sqrt{s_t' \hat{W} s_t}$).

W_t , to be time-dependent. In particular, we adopt the state-space system

$$x_t | W_t \sim N(c_t + \Phi_t x_{t-1}, W_t), \quad (2)$$

$$W_t | W_{t-1} \sim \text{WAR}(1), \quad (3)$$

where (2) is the observation equation and (3) the transition equation for the covariance matrix W_t . The vector of constants is denoted by c_t and Φ_t is the diagonal matrix with elements ϕ_t^i . Sectoral interactions through autoregressive components are excluded, based on the empirical findings of Neusser (2008). The coefficients are indexed by time to allow for the breaks detected in Section 2.2.

Following the seminal contribution of Gouriou et al. (2009), we model the covariance matrices (W_t) as a Wishart process of order one (WAR(1) process). The WAR(1) process is an autoregressive process for covariance matrices with given dimension n_s . Its moment-generating function, $\mathcal{M}(\Gamma)$, is

$$\mathcal{M}(\Gamma) = \mathbf{E}_t[\exp(\text{Tr}(\Gamma W_{t+1}))] = \frac{\exp(\text{Tr}[M' \Gamma (I_{n_s} - 2\Sigma \Gamma)^{-1} M W_t])}{[\det(I_{n_s} - 2\Sigma \Gamma)]^{K/2}},$$

where \mathbf{E}_t is the expectation operator conditional on information up to time t and Tr

the trace operator. The $n_s \times n_s$ matrix M comprises the autoregressive parameters of the process, and Σ is on $n_s \times n_s$ symmetric and positive definite matrix. The degree of freedom K is a positive number satisfying $K > n_s - 1$. I_{n_s} is the identity matrix of dimension n_s . The moment-generating function is defined for symmetric matrices Γ with $\|2\Sigma\Gamma\| < 1$.⁸

Assuming that the degree of freedom K is an integer, the process can be rewritten in an instructive way as

$$W_t = \sum_{k=1}^K z_{k,t} z'_{k,t},$$

where (z_{kt}) , $k = 1, \dots, K$, are independent Gaussian vector processes of dimension n_s that satisfy

$$z_{k,t} = M z_{k,t-1} + e_{k,t}, \quad e_{k,t} \sim \text{IID } N(0, \Sigma).$$

If $K = 1$, $M = 0$, and $\Sigma = 1$, one can recognize the $\chi^2(1)$ distribution as a special case of the Wishart distribution. Observe that the stochastic matrix W_t is of full rank with probability one if the degree of freedom, denoted by K , is equal to or greater than n_s .

It is instructive to analyze the behavior of (W_t) as the degree of freedom, K , goes to infinity. Obviously, if the covariance matrix Σ of the $e_{k,t}$ vectors is independent of K , (W_t) has an exploding second moment matrix as $K \rightarrow \infty$. However, we can consider the transformed processes $\tilde{z}_{k,t} := \sqrt{K} z_{k,t}$ and $\Sigma(K) = K^{-1} \tilde{\Sigma}$, where $\tilde{\Sigma}$ is a constant matrix, and write

$$W_t = \frac{1}{K} \sum_{k=1}^K \tilde{z}_{k,t} \tilde{z}'_{k,t}, \quad (4)$$

where

$$\tilde{z}_{k,t} = M \tilde{z}_{k,t-1} + \tilde{e}_{k,t}, \quad \tilde{e}_{k,t} \sim \text{IID } N(0, \tilde{\Sigma}). \quad (5)$$

If all eigenvalues of M are strictly inside the unit circle, the unconditional distribution of $\tilde{z}_{k,t}$ is $N(0, \tilde{\Sigma}(\infty))$, where $\tilde{\Sigma}(\infty)$ solves

$$\tilde{\Sigma}(\infty) = M \tilde{\Sigma}(\infty) M' + \tilde{\Sigma}.$$

Given a sample $\{\tilde{z}_{k,t} \tilde{z}'_{k,t}\}_{k=1}^K$ of identically and independently distributed random variables with finite second-order moments, the strong law of large numbers implies that the sum in equation (4) converges almost surely to $\mathbf{E}[\tilde{z}_{k,t} \tilde{z}'_{k,t}]$ as $K \rightarrow \infty$. Therefore, in the limit as $K \rightarrow \infty$, the WAR(1) process is a degenerated process with constant matrices as its realizations. We will use this fact to calibrate the process (Section 3.3).

Note that the Wishart specification is conceptually quite different from the class of multivariate generalized ARCH (MGARCH) models:⁹

- (i) In a MGARCH model, the volatility of x_{t+1} conditional on information up to time t depends on past realizations of growth rates. This is not the case for the state-space system with the Wishart specification.
- (ii) The estimation of volatility matrices in a MGARCH framework requires estimating a large number of parameters. Even the multivariate ARCH(1) model, given by

$$\text{vech}(\Sigma_t) = b + A \text{vech}((x_t - c_t - \Phi_t x_{t-1})(x_t - c_t - \Phi_t x_{t-1})'),$$

involves

$$\left(\frac{n_s(n_s + 1)}{2}\right)^2 + \frac{n_s(n_s + 1)}{2}$$

parameters.¹⁰ The literature has proposed to put restrictions on the parameters [see, for example, Engle and Kroner (1993)], which, however, can be “*complicated and hard to interpret*”¹¹ because they have to be such that the conditional volatility matrices are positive definite, whereas the WAR(1) model requires only

$$n_s^2 + \frac{n_s(n_s + 1)}{2} + 1$$

parameters.

3.2. Particle Filter

The observation equation (2) and the transition equation (3) form a state-space system. Because of its nonlinearity, the Kalman filter cannot be applied. Therefore the particle filter (sequential Monte Carlo filter) is used to draw from the posterior distribution of the covariances W_t given the observed growth rates up to time t . For a detailed discussion on particle filtering methods, we refer to Arulampalam et al. (2002).¹² The parameters of the model, M , Σ , and K , are subsumed into θ and the density of W_t given the observed growth rates up to date t is denoted by $p(W_t|x_{1:t}, \theta)$. $f_x(x_t|W_t)$ refers to the density of x_t given W_t , and $f_w(W_t|W_{t-1}, \theta)$ to the transition density of the WAR(1). Given this notation, the particle filter consists of the following three steps:

Step 1. Initialization. Draw N times from the unconditional distribution of W_0 , denoted by $p(W_0|\theta)$. This gives a sample of N particles, which is denoted by $\{w_{0|0}^i\}_{i=1}^N$.

Step 2. Prediction. For each of the particles in the sample from the previous step, given by $\{w_{t-1,t-1}^i\}_{i=1}^N$, generate one draw from $f_w(W_t|w_{t-1,t-1}^i, \theta)$. The result is a sample of N particles, $\{w_{t|t-1}^i\}_{i=1}^N$, that are drawn from $p(W_t|x_{1:t-1}, \theta)$.

Step 3. Updating. Draw, with replacement, N times from the previous sample, $\{w_{t|t-1}^i\}_{i=1}^N$. The probability of drawing $w_{t|t-1}^i$ is given by the so-called normalized importance weight, denoted by π_t^i , which is calculated according to

$$\pi_t^i = \frac{f_x(x_t | w_{t|t-1}^i)}{\sum_{i=1}^N f_x(x_t | w_{t|t-1}^i)}.$$

The resulting sample is a draw from the discretized density $p(W_t | x_{1:t}, \theta)$.

3.3. Implementation

We implement the Wishart model based on the following assumptions. First, the autoregressive matrix M is restricted to $M = \sqrt{\lambda} I_{n_s}$, with λ equal to 0.8.¹³ The exclusion of interactions among sectors through the autoregressive matrix M can be justified by the evidence presented in Neusser (2008), although the context and the data are not comparable. He does not find a sensible and significant intersectoral covariance structure of innovations in his analysis of sectoral total factor productivity (TFP) growth. Second, we set K equal to the number of sectors, i.e., 19.¹⁴ This choice is motivated by the fact that a value for K less than 19 would lead to singular covariance matrices and that values greater than 19 would result in more persistent covariance matrices, which in turn would magnify the role played by sectoral shifts. Third, we determine Σ by setting $K\Sigma$ equal to the sample covariance matrix of the data multiplied by $(1 - \lambda)$. This calibration is justified by the convergence of W_t to $\tilde{\Sigma}(\infty)$. If $\tilde{\Sigma}(\infty)$ is assumed to be close to the sample covariance matrix, equation (5) implies that $K\Sigma = \tilde{\Sigma} = (1 - \lambda)\tilde{\Sigma}(\infty)$.

The particle filter is run 20 times, drawing 20,000 particles each time. Then we average the 20 estimates of $\mathbf{E}[W_t | x_{1:t}]$ and take this as an estimator for the standard deviation. We initialize the filter by setting $z_{k,0} = x_k - c_k - \Phi_k x_{k-1}$, where k runs over the first nine years of our sample.

Although it is known that the particle filter can suffer from a degeneracy problem, i.e., that after a few iterations, only one particle gets through the filter, we did not encounter this problem. As an illustration, Figure A.1 in Appendix A shows the 2005-posterior distributions of all covariance matrix elements associated with durable goods production.

4. RESULTS

Comparing the results of the ICSS variance break tests in Table 3 with Figures 8.1–8.4 in Appendix B shows that the filtered standard deviations of mining, utilities, durable goods, and information square well with the findings of the ICSS variance break test.¹⁵ This indicates that the evidence produced by the WAR model is sensible.

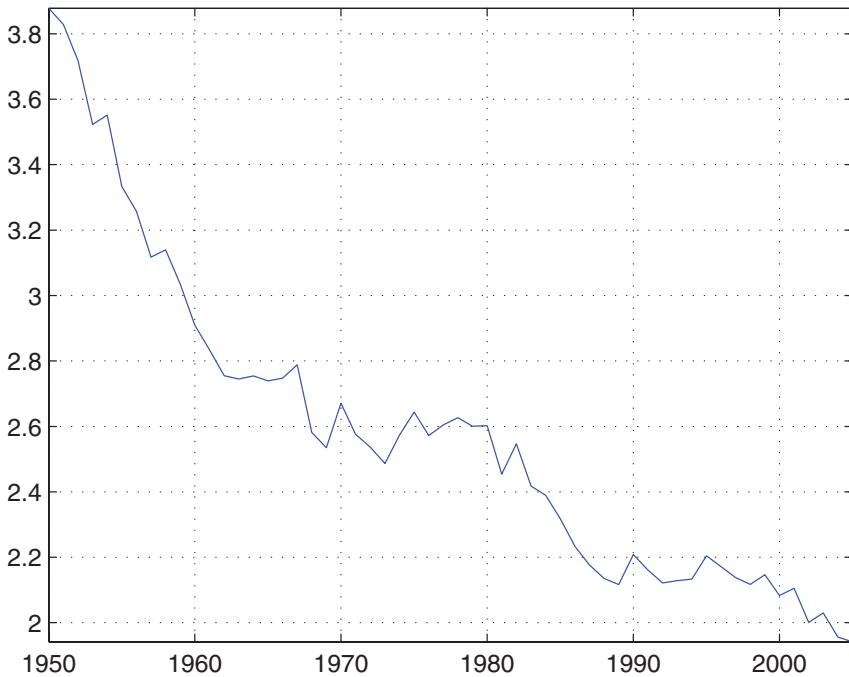


FIGURE 3. Conditional standard deviation of GDP.

4.1. Conditional GDP Volatility

We use the filtered covariance matrices to assess the dependence of the conditional standard deviation of GDP growth on sectoral shares. The variance of real GDP growth rates $\Delta y_t / y_{t-1}$ is formally given as

$$\mathbf{V}_{t-1} \left(\frac{\Delta y_t}{y_{t-1}} \right) = \mathbf{V}_{t-1} (\zeta'_{t-1} x_t) = \zeta'_{t-1} W_t \zeta_{t-1} = \sum_{i=1}^{22} \sum_{k=1}^{22} (\zeta_{t-1})_i (\zeta_{t-1})_k (W_t)_{ik}, \quad (6)$$

where y_t and $\zeta_{t-1} = ((\zeta_{t-1})_1, \dots, (\zeta_{t-1})_{22})'$ denote real GDP and the vector of sectoral shares, respectively. The conditional GDP standard deviation is just the square root of this expression. Figure 3 plots this entity for the period 1949–2005. We can see that it decreased more or less gradually from around 3.8 to less than 2 and therefore supports the trend-decline hypothesis expressed by Blanchard and Simon (2001). However, this downward trend speeds up in the early 1950s and the mid-1980s. Because of endpoint issues, the Korean war (e.g., the recovery from the 1949–1950 recession had 13% growth over four quarters), and the data quality in the early 1950s, we focus our attention on the latter downward acceleration, which corresponds to what is labeled in the literature as the Great Moderation.

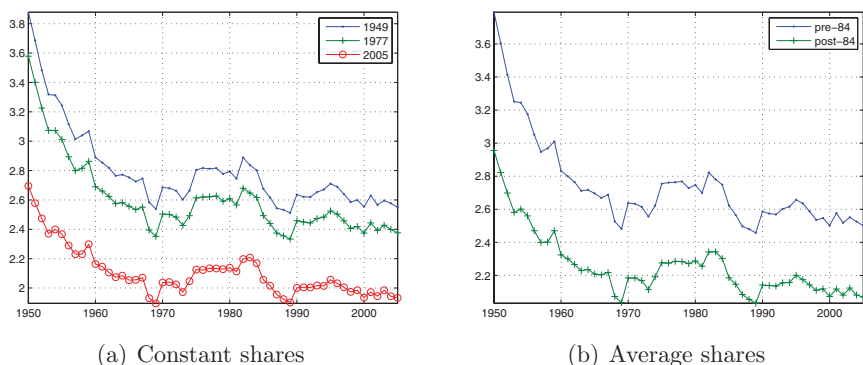


FIGURE 4. Conditional standard deviation of GDP.

To get an idea of how the composition of output affects its volatility, we compute three counterfactual paths for the conditional GDP standard deviations. These paths are obtained by evaluating equation (6) in each period, fixing the shares at their values in 1949, 1977, and 2005, respectively, but using the same (W_t) -sequence. Figure 4a plots these paths. There is an unambiguous ranking: the conditional GDP standard deviation with the 2005 shares is on average 24.7% (19.1%) lower over the whole sample than the one with 1949 shares (1977 shares). We get a similar finding if we set the shares to their average pre-1984, respectively post-1984 averages, as in Figure 4b. The standard deviation with post-1984 shares is on average 17.7% lower than pre-1984 values. This suggests that a non-negligible part of the decline in the conditional GDP standard deviation can be attributed to sectoral shifts. The findings are robust if a less persistent $\text{WAR}(1)$ is assumed.¹⁶

4.2. Unconditional GDP Volatility

We use simulations to infer the unconditional GDP volatility for given initial (1949) sectoral shares. For this purpose, we use the $\text{AR}(1)$ models from Section 2.2 to simulate sectoral production, taking the parameter breaks into account. We draw the innovations from multivariate normals with covariance matrices equal to the filtered W_t .¹⁷ We further compute the standard deviation of the implied GDP growth rate for the pre-1984 and the post-1984 periods. We repeat this exercise 100,000 times and finally compute the average standard deviations that are reported in Table 5 for alternative initial shares. The simulation with the initial shares from 1949 can serve as a check of our specification. In particular, we verify if GDP volatility falls inside the intervals [2.06, 3.24], respectively [1.38, 2.62], whose bounds correspond to the 5 and 95% quantiles of the pre-1984, respectively post-1984 period. As GDP volatility is 3.21 in the pre-1984, respectively 1.63, in the post-1984 sample, we feel comfortable with our specification.

Table 5 reveals that the average simulated standard deviations depend on the chosen initial sectoral shares. The simulated standard deviation for the 2005 (1977)

TABLE 5. Average GDP standard deviation

Initial shares	1950–1984	1985–2005
1949	2.62	1.95
1977	2.37	1.75
2005	1.95	1.63

initial shares is 25.6% (17.7%) lower in the pre-1984 period than the simulated standard deviation for the 1949 initial shares. For the post-1984 period, the respective percentage is 16.4% (10.3%).¹⁸

4.3. Relevant Sectoral Shifts

In the previous sections, we have found that GDP volatility depends, to a significant and sizable extents, on its sectoral composition. The purpose of this section is to identify the relevant sectoral changes behind this relationship.

The five most important sectoral shifts—in absolute value and in order of decreasing importance—took place in the sectors durable goods, nondurable goods production, finance and insurance, professional—scientific and technical services, and health care and social assistance.¹⁹ The numbers in Table 6 show that both durable and nondurable goods production have become less important than the three service sectors finance and insurance, professional—scientific and technical services, and health care and social assistance. In the years 2001–2005, these three service sectors made up over 20% of real GDP.

Figure 5 plots the filtered standard deviations for these five sectors as computed from (W_t) . This chart shows that the two production sectors indeed have higher standard deviations than the three service sectors. This evidence thus confirms our hypothesis that the sectoral shift indeed had a dampening effect on GDP volatility.

A possible drawback of the preceding analysis is that it does not take the evolution of covariance terms into account. We therefore use an alternative method to identify the relevant sectoral shifts. For this purpose we compute the ik th summand in the double sum defining the conditional variance of real GDP growth, $\mathbf{V}_{t-1}(\frac{\Delta y_t}{y_{t-1}})$, which, according to equation (6), is equal to $(\zeta_{t-1})_i(\zeta_{t-1})_k(W_t)_{ik}$. This

TABLE 6. Average shares (in percent)

Sector	Average share 1948–1953	Average share 2001–2005
Durables	17.86	8.39
Nondurables	15.04	6.14
Finance and Insurance	3.27	9.19
Professional—scientific and technical services	1.93	7.84
Health care and social assistance	2.02	7.83

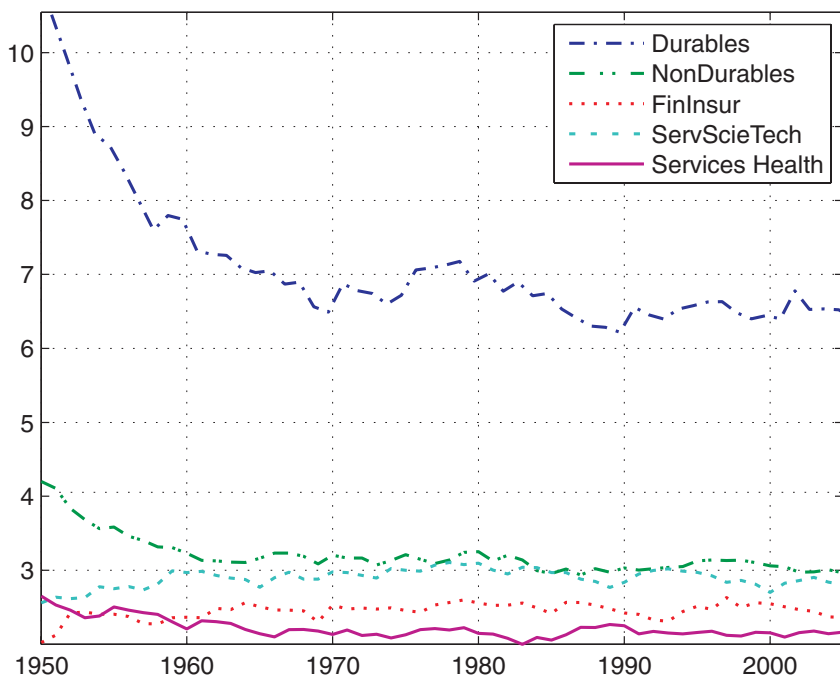


FIGURE 5. Conditional standard deviations of sectors with largest shifts.

analysis takes the path of the entire covariance matrices (W_t) into consideration. It reveals that the variance corresponding to the durable goods sector exhibits the largest decrease. Other significantly decreasing terms correspond, in order of descending magnitude, to

- (a) the covariance between durable and nondurable goods production,
- (b) the covariance between durable and retail sales,
- (c) the covariance between durable goods and wholesale trade, and
- (d) the variance of nondurable goods production.

Figure 6 shows a plot of these covariances.

To disentangle the changes in the shifts from the changes in the covariance matrix, we also computed $\zeta_{t-1}^i \zeta_{t-1}^k (\bar{W})_{ik}$, where \bar{W} is the average covariance matrix over the entire sample 1949–2005. We find again that the variance term of durable goods production exhibits the largest decrease. In addition, we find significant decreases for

- (a) the covariance term between durable and nondurable goods,
- (b) the covariance term between durable goods and retail trade,

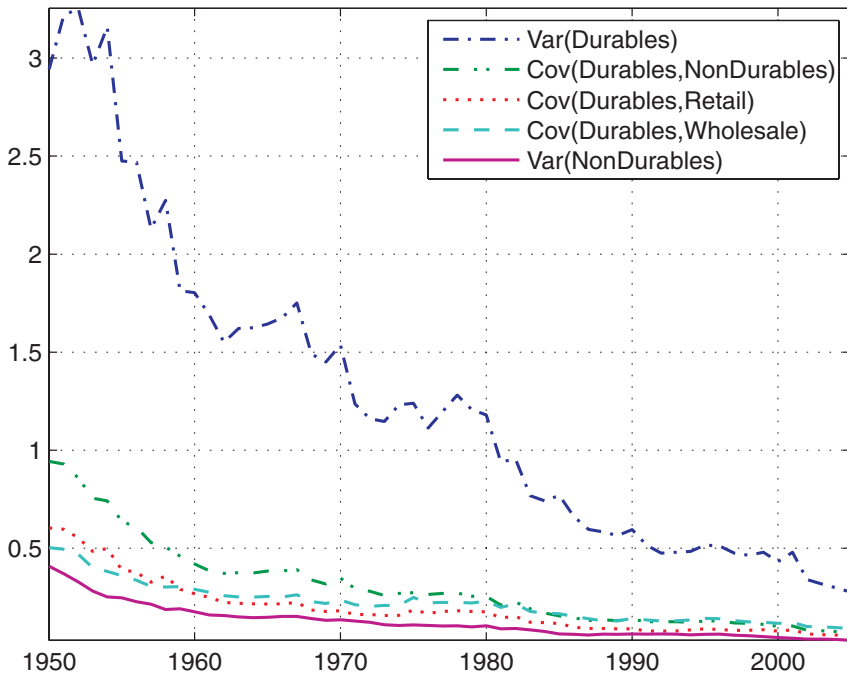


FIGURE 6. Five most important $\zeta_{t-1}^i \zeta_{t-1}^k (W_t)_{ik}$ -terms.

- (c) the variance term of nondurable goods, and
- (d) the covariance term between durable goods and wholesale trade.

These variances and covariances are plotted in Figure 7.

We conclude that the shift out of durable goods production has significantly stabilized real GDP growth. Not only a decrease in the variance term, but also the decrease in covariance terms, led to this stabilization.

5. CONCLUSION

We investigated whether and to what extent the decline in the volatility of US GDP, observed for the period 1949 to 2005, can be attributed to the structural change in its sectoral composition. We found that structural changes alone would account for approximately 30% of the observed moderation. This evidence was produced from a Wishart autoregressive model that allows for time-varying covariances. Running some counterfactual experiments, we can decompose the effect of structural changes into a pure sectoral change, i.e., changes in the sectoral shares, and a change in sectoral volatility. These experiments indicate that, even if covariance matrices are held constant, pure sectoral changes explain 15% of the decline in GDP volatility. In addition, the conditional standard deviation of GDP growth would have been, on average, 24.7% lower if sectoral shares in 1949 had been

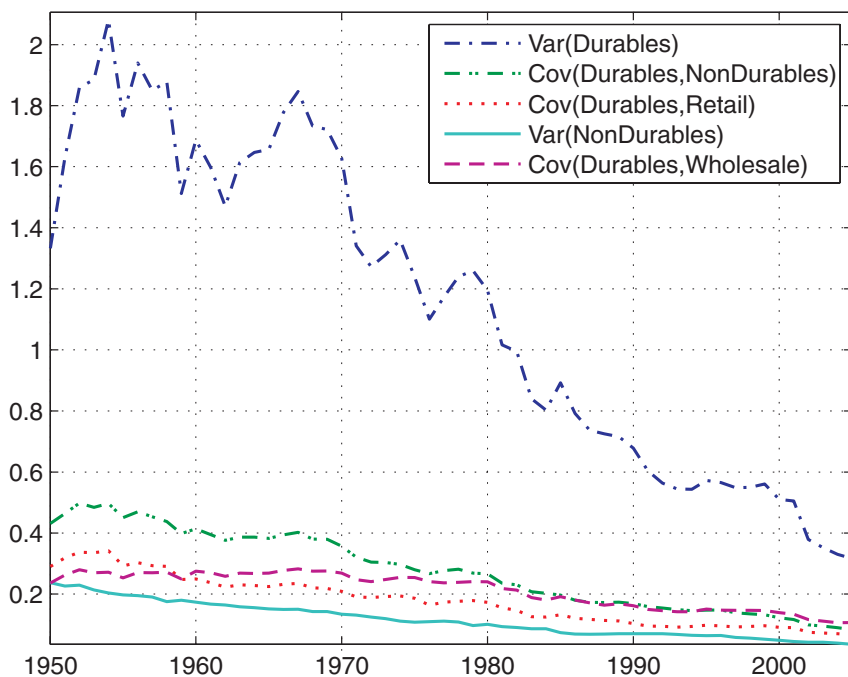


FIGURE 7. Five most important $\zeta_{t-1}^i \zeta_{t-1}^k (\bar{W})_{ik}$ -terms.

equal to those in 2005. For the unconditional standard deviation, the reduction would have been 25.6%.

This evidence is driven to an important extent by the reduction in the share of durable goods production. This result is well in line with those of McConnell and Perez-Quiros (2000), who argue that the reduction in the variance of durable goods production alone can account for the break in GDP volatility. Furthermore, the increased openness of the U.S. economy is consistent with this reasoning, as it made it possible to outsource production, in particular durable goods production, to other countries.²⁰ However, the cumulative sums-of-squares test of Inclán and Tiao (1994) showed that durable goods production is the only sector with a significant reduction in its conditional variance. Some sectors, such as mining, utilities, and information, even experienced an increase. These findings cast some doubt on the Good Luck hypothesis.

Many of the existing economic models, which focus on business cycles, shut out the growth component. However, as we have shown, this leaves out an important part of the story. Therefore, future research should aim at developing behavioral models that rationalize the observed sectoral shifts and link economic growth and volatility. Promising sources of inspiration could be Bernanke (1983) and Pindyck (1991). They argue that the option character makes irreversible investment decisions prone to risk. Increased uncertainty pushes the investor to postpone projects

and wait for new information about prices, costs, and other market conditions. This leads to a negative relationship between growth and volatility. Similar arguments can be put forward in the case of human capital accumulation [see Martin and Rogers (1987) or Blackburn and Galindez (2003)].

NOTES

1. Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) provide the first published expositions of this fact.
2. For further properties and a deeper discussion we refer to the original article by Gouriou et al. (2009). This paper also provides additional justifications for this approach, in particular with regard to multivariate GARCH models.
3. The implicit GDP deflator is from the BEA and is given by the ratio of the GDP is current-dollar value to its chained-dollar value.
4. Agriculture had negative average growth. We took its absolute value.
5. Except for the sectors agriculture, health care and social assistance, and federal government, the AR(1) structure is sufficient to characterize the data. Evidence on this point can be obtained from the authors.
6. The ICSS test also indicates significant volatility breaks for agriculture (the one period–forecast standard deviation increased from 5.90 to 13.75 in 1972) and federal government (the conditional standard deviation decreased from 14.70 to 2.13 in 1953). However, significant autocorrelation in the residuals violates the assumptions of the ICSS test. Thus the results for these two sectors are not reliable.
7. It also indicates a significant decrease in federal government variance; however, this could be spurious, because there is significant correlation left in these residuals.
8. $\| \cdot \|$ denotes the operator norm. The submultiplicative property of the operator norm implies that the condition is satisfied if the product of the two largest eigenvalues is less than 0.5.
9. Engle and Kroner (1993) and Tse and Tsui (2002) discuss MGARCH models.
10. The operator *vech*(.) stacks the different elements in Σ ; because there are

$$n_e := \frac{n_s(n_s + 1)}{2}$$

such different elements, there are n_e equations each involving n_e regressors, which gives the n_e^2 parameters in A.

11. See Gouriou et al. (2009).
12. These methods have been used in economics by An (2005) and by Fernandez-Villaverde and Rubio-Ramirez (2007) to estimate a second-order approximation to dynamic stochastic general equilibrium models (DSGE models).
13. Other values of λ lead to the same conclusions regarding the contribution of sectoral shares in GDP volatility. A higher λ means that the Wishart is more persistent and has weaker innovations.
14. Agriculture, federal government, and state and local government are left out.
15. Graphs of filtered standard deviations of other series, as well as graphs of filtered covariances, are available from the author upon request.
16. Further counterfactual calibrations are available from the authors upon request.
17. Because the AR(1) parameters are different, sectors grow at different rates in our simulations, and therefore, shares depart from their initial values. We did not choose to calibrate AR(1) parameters such that shares would stay, on average, close to their initial values. The reason is that there are many different possible calibrations that can do this and that have, by themselves, different implications on simulated GDP volatility.
18. The results do barely change with different Wishart calibrations.

19. The absolute difference of the average shares in the years 1948–1952 and the average shares in the years 2001–2005 is computed. The list hardly changes if, instead, the absolute difference between pre-1984 and post-1984 average shares is taken.

20. See Barrell and Gottschalk (2004), who relate the increased openness to the decline in U.S. output volatility.

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APPENDIX A: FILTERED COVARIANCES OF DURABLES

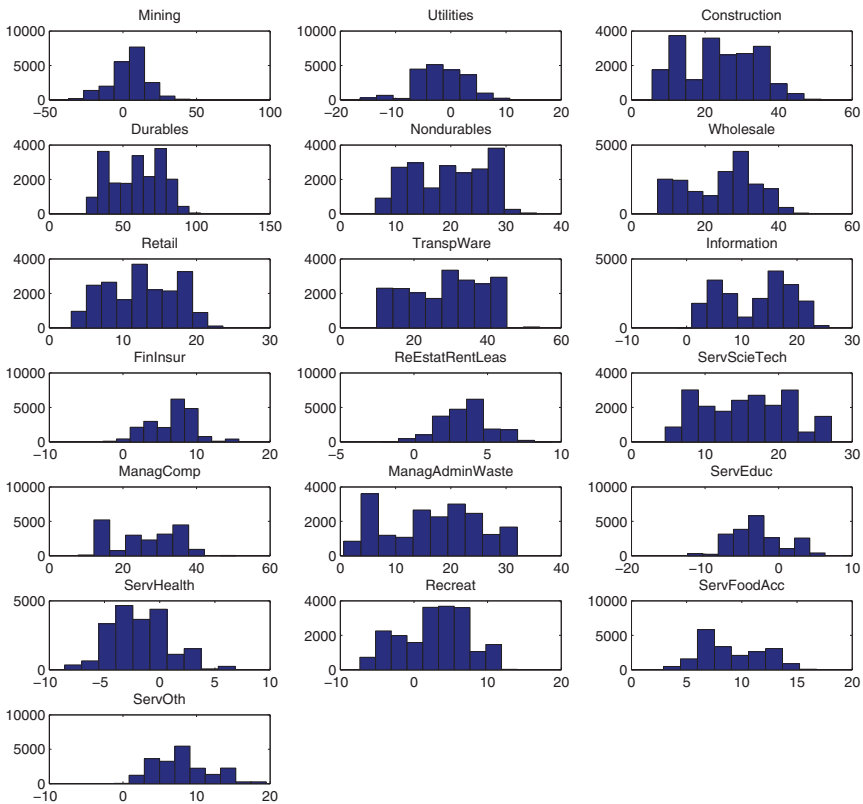
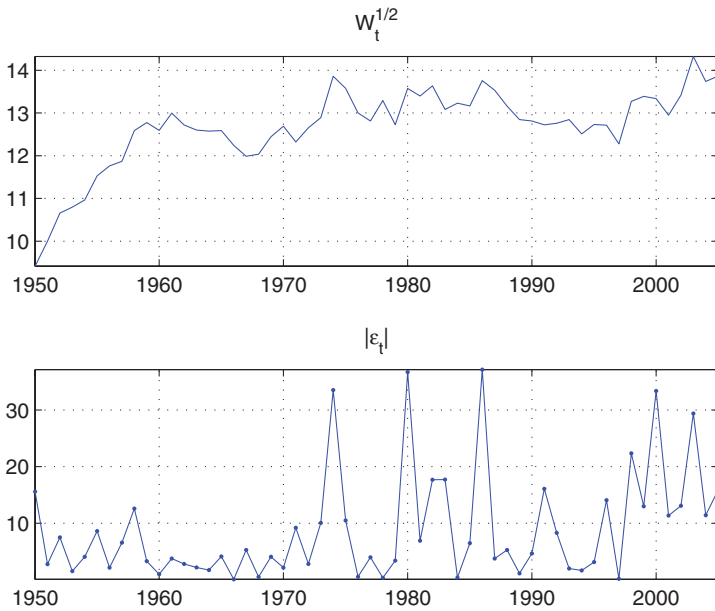


FIGURE A.1. Filtered covariances between durables and other sectors.

APPENDIX B: FILTERED STANDARD DEVIATIONS

**FIGURE B.1.** Filtered standard deviations for the mining sector.

In order to save space, only the filtered standard deviations for mining, utilities, durable goods, and information are reported (Figures B.1–B.4). All other graphs are available upon request. For these four sectors, the ICSS test detected breaks in the conditional variance (see Table 3). The upper graph shows the filtered standard deviations. The bottom graph plots the absolute values of the residuals from the AR(1) regressions.

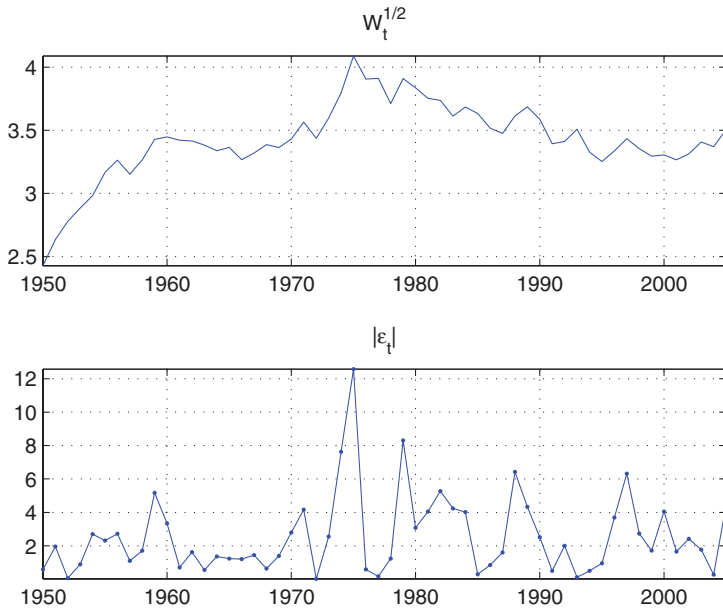


FIGURE B.2. Filtered standard deviations for the utilities sector.

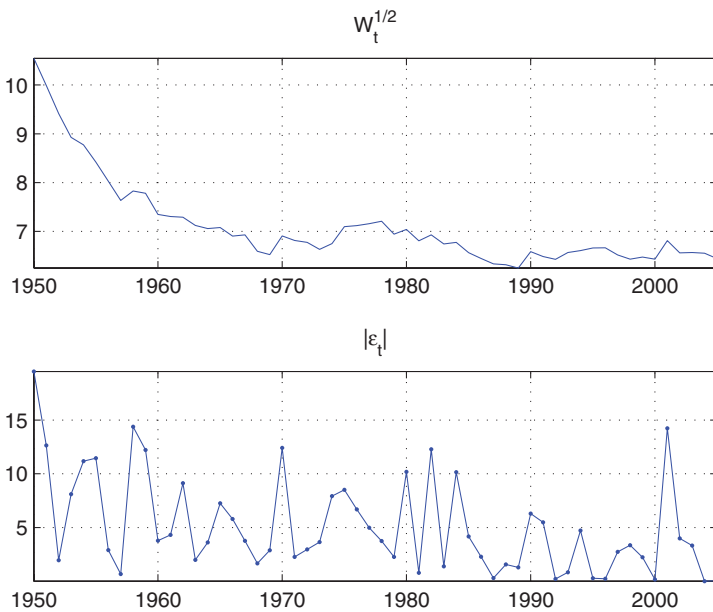


FIGURE B.3. Filtered standard deviations for the durables sectors.

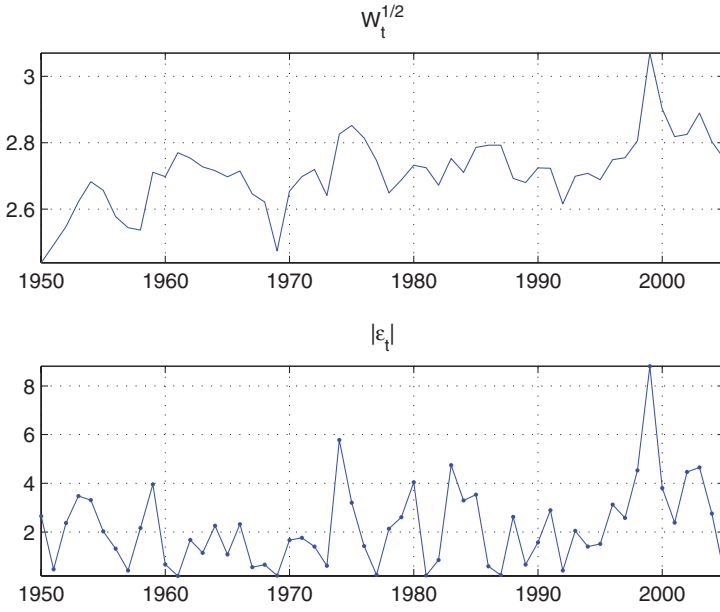


FIGURE B.4. Filtered standard deviations for the information sector.